

on the epistemological significance of arguments from non transitive similarity

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Abstract

This paper aims to argue for, else illustrate the epistemological significance of the use of non transitive similarity relations, mapping only to "types"¹, as methodologically being on a par with the use of transitive similarity relations (equivalence relations), mapping as well to "predicates".

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¹a "type", for short, is given by a non empty set of paradigms and a (transitive or non transitive) similarity relation, relating at least the paradigms in some way.

1 Introduction

The well known basic fact, that an equivalence relation scatters (splits up) its domain into a set of mutually disjoint equivalence classes, while a non transitive similarity relation does not, is here of central importance. The point is hence, that, because equivalence classes are not available for non transitive similarity relations, a non transitive similarity relation may not be taken to generate a predicate (neither an unary nor in the general case an n-ary predicate), while a transitive does.²

Thus, use of types with their tokens, of paradigms with their implementations, ... while logically and hence methodologically unobjectionable, is not in general equivalent to the use of predicates in the standard sense; i.e.: predicates [with a sharply determined extension] are only a very special case of types, viz. those determined by a transitive similarity relation.

Readers not used to formal considerations might skip section(2), viz. the recapitulation of an exact model of the above construct, and start reading with section(3); though not without loss of content, as the model - which is an extremely simple geometric one - does not only allow to map available similarity comparisons and/or similarity measurements to the model, but allows also in some obvious way for visualization of the mapped similarity information.

In section(3), use cases for non transitive similarity relations from science and humanities are mentioned, in section(4) a metaphysics example is expanded to some extent.

2 similarity relations: definition and trigonometric interpretation - a short recapitulation

In my recent 'note on Sorites series' ([11] section 2, pp.2-4), a very general and straightforward definition of similarity relations was used (2-place relations, reflexive, symmetric, may or may not be transitive), thus having equivalence relations as a special case ([11] section 2.1), and I introduced an "imagination supporting" elementary trigonometric model in Euclidean space. This model consists of a domain of line segments of equal length and arbitrary directions, the domain is closed with respect to linear and parallel translation (rotation does not belong to the model, Euclidean distance does). "similarity" between 2 objects of

²A predicate is said to be *generated* from an equivalence relation, *iff* the union of some equivalence classes is taken as the extension of the predicate, and the union of the remaining equivalence classes as the complement of the predicate's extension in the domain. On the other hand, any n-ary predicate $\mathbf{P}\mathbf{x}_1, \dots, \mathbf{x}_n$ is accompanied by the trivial equivalence relation $\mathbf{P}\mathbf{x}_1, \dots, \mathbf{x}_n \leftrightarrow \mathbf{P}\mathbf{y}_1, \dots, \mathbf{y}_n$, especially in the unary case $\mathbf{P}\mathbf{x}$ is accompanied by $\mathbf{P}\mathbf{x} \leftrightarrow \mathbf{P}\mathbf{y}$.

Less common, but useful for generalization in talk of similarity relations is the following observation: to any predicate \mathbf{P} there exists its *type* $\mathbf{T}_\mathbf{P}$, coinciding with the predicate's extension $\overline{\mathbf{P}}$: from any equivalence class, whose union is the predicate's extension $\overline{\mathbf{P}}$, select one member, they altogether forming the paradigm set $\mathbf{PD}_\mathbf{P}$. The type $\mathbf{T}_\mathbf{P}$ is then specified as the smallest set, including any object similar to an element of the paradigm set $\mathbf{PD}_\mathbf{P}$, where the (in this case transitive) similarity relation is simply the equivalence relation, that creates the equivalence classes.

the domain (= 2 line segments) in this model is pictured by the smallest angle α , these 2 line segments include as isosceles, when suitably mapped by linear and parallel translations. Obviously this smallest angle is $0^\circ \leq \alpha \leq 90^\circ$. Two objects (isosceles line segments) are (more or less) similar, if $0^\circ \leq \alpha \leq 60^\circ$ [³], otherwise (more or less) not similar ([11] section 2.2). The model does serve not only as a model of comparative similarity as just described, but also as a model for similarity by degree ([11] section 2.3). While I did not explicitly state in the originating paper, a natural choice for the respective similarity measure⁴ might of course be $\mathbf{cos} \alpha$ for being monotone in the first quadrant, and in case $\alpha = 0^\circ$ (isosceles line segments coincide), then $\mathbf{cos} \alpha = \mathbf{1}$, and in case $\alpha = 90^\circ$, then $\mathbf{cos} \alpha = \mathbf{0}$ [⁵]; additionally welcome is $\mathbf{cos} 60^\circ = \mathbf{0,5}$ [⁶]. Lastly ([11] section 2.4.1) gives a general formula for a Sorites series $\mathbf{Fa}_1, \dots, \neg\mathbf{Fa}_n$ chained by a non transitive 2-place similarity relation S , viz.

$$\mathbf{F a}_1 \wedge \mathbf{S a}_1, \mathbf{a}_2 \wedge \dots \wedge \mathbf{S a}_{i-1}, \mathbf{a}_i \wedge \mathbf{S a}_i, \mathbf{a}_{i+1} \wedge \neg\mathbf{S a}_{i-k}, \mathbf{a}_{i+1} \wedge \neg\mathbf{F a}_{i+1} \dots$$

3 use cases for (mostly non transitive) similarity relations

3.1 historical and other preliminary remarks

What we are about here, is to strive for the recognition of non transitive similarity as a logically consistent and scientifically fully legitimate logical means for the study of typologies/taxonomies, cluster phenomena, some sorts of claimed vagueness of concepts, and the like, without any need to resort to whatever kind of non standard logic (modal, many-valued, para-consistent, ...) for this purpose.

The related explicit methodological discussion historically started with the groundbreaking debate between William Whewell and John Stuart Mill on whether 'natural groups' are 'given by type, not by definition', referring to the botanic type example 'the rose family(=rosaceae)'⁷. This debate - while not in

³in case $\alpha = 0^\circ$ the isosceles coincide, hence the isosceles triangle collapses into the isosceles line segment, in case $\alpha = 60^\circ$ the isosceles triangle is also an equilateral triangle, thus the length of the opposite side is equal to the isosceles' length. And in case $\alpha = 90^\circ$ the length of the opposite side of the then right-angled isosceles triangle is of course the square root of 2, if the length of the isosceles equals 1 (, as we can always assume).

⁴In statistics and related fields, a similarity measure or similarity function is a real-valued function that quantifies the similarity between two objects. Although no single definition of a similarity measure exists, usually such measures are in some sense the inverse of distance metrics: they take on large values for similar objects and either zero or a negative value for very dissimilar objects." wikipedia article "similarity measure" [23], p.1

⁵in [11] I referred to respective axioms T1-T5 from Timothy Williamson's [24], pp.461f., and its obvious, that $\mathbf{cos} \alpha$ in the first quadrant satisfies Williamsons requirements for a respective similarity measure $M1 - M4$ op.cit. p.459

⁶this effect depends of course on the deliberate choice of $\mathbf{cos} \alpha$ as the similarity measure (helping again for visualization). Else other monotone mappings to $[\mathbf{1}, \mathbf{0}] \subset \mathbb{R}$ could serve as a similarity measure as well.

⁷the most crucial passages at Whewell [18] Chapt. II,§9 Difference of Natural History and Mathematics, and §10 Natural Groups given by Type, not by Definition, pp. 121 f., and Mill [13],Chapt. VII, § 4, pp. 278ff., especially p.282, have been cited in my 'note on Sorites series'

the least having been settled - to my best knowledge was discontinued thereafter. In the sequel, it seems that the rapid development of formal logic and logic based philosophy of science throughout the twentieth century had plowed this sensitive epistemological topic, which - presumably for alleged epistemic inferiority - remained mostly outside the attention of logical empiricism. Discussions on 'natural properties' starting say from the fifties onward, e.g. Nelson Goodman's "grue", while rightly reminding us, that the predicates (and for that, the similarity relations) considered should prove some inductive value⁸, does not directly touch on the debate, of 'given by type vs. given by definition'. And also Quine, in his lecture on 'Natural Kinds', while pointing convincingly to the intimate correlation between natural kinds and similarity, does not pay special attention to non transitive similarity. His considering natural kinds as 'sets' does not differentiate between transitive and non transitive similarity⁹. And he strictly judges use of natural kinds in science as epistemically inferior¹⁰. Again from late discussion, the assumption of 'essential' else 'intrinsic' properties (for belonging to a species), see e.g. Richard Boyd [4], seems somewhat question begging in our context, at least as far as this assumption is meant to imply/presuppose the possibility of natural groups generally 'given by definition'.

Now, of course, lots of everyday judgments and perceptions are based on, or presuppose, judgments of (dis)similarity, and as well to a considerable extent even scientific judgments do¹¹, and in the age of industry and information technology also lots of technical processes depend on determination of (dis)similarities, and one might have the impression, that in almost all significant cases, the similarity relations involved concern domains, in which they

⁸[10] pp. 72-81

⁹thus Quine considers the proposal "If without serious loss of accuracy we can assume that there are one or more actual things (paradigm cases) that nicely exemplify the desired norm, and one or more actual things (foils) that deviate just barely too much to be counted in the desired kind at all, then our definition is easy: the kind with paradigm a and foil b is the set of all the things to which a is more similar than a is to b. More generally, then, a set may be said to be a kind if and only if there are a and b, known or unknown, such that the set is the kind with paradigm a and foil b." [16], pp. 119f.

¹⁰In general, we can take it as (as we can always assume) a very special mark of the maturity of a branch of science that it no longer needs an irreducible notion of similarity and kind. ... In [this] career of the similarity notion, starting in its innate phase, developing over the years in the light of accumulated experience, passing them from the intuitive phase into theoretical similarity, and finally (as we can always assume) appearing altogether, we have a paradigm of the evolution of unreason into science" [16], p. 138

¹¹there is an important remark of Whewell in the already cited context, on the coining and the reliability of scientific knowledge generated with use of "types", and here is the full text of this remark:

"11. It has already been repeatedly stated, as the great rule of all classification, that the classification must serve to assert general propositions. It may be asked *what* propositions we are able to enunciate by means of such classifications as we are now treating of. And the answer is, that the collected knowledge of the characters, habits, properties, organization, and functions of these groups and families, as it is found in the best botanical works, and as it exists in the mind of the best botanists, exhibits to us the propositions which constitute the science, and to the expression of which the classification is to serve. All that is not strictly definition, that is, all that is not artificial character, in the description of such classes, is a statement of truths, more or less general, more or less precise, but making up, together, the positive knowledge which constitutes the science. As we have said, the consideration of the properties of plants in order to form a system of classification, has been termed Taxonomy, or the Systematick of Botany; all the parts of the descriptions, which, taking the system for granted, convey additional information, are termed the *Physiography* of the science; and the same terms may be applied in the other branches of Natural History." ([18] Chapt. II, §11, pp. 122f.)

are not in general transitive.

Although we are really well acquainted with the ubiquitous presence of similarity judgments, let's just mention some use cases, to recall the epistemic flavor of that 'knowledge by acquaintance' of uses of similarity, as we here intend a bit of 'knowledge by description' of them, to borrow for sake of shortness from Russell's terminology

3.2 mentioning use cases for similarity by degree

Similarity by degree is of course not restricted to non transitive similarity relations, but these, I think, are well expected to make the prominent use cases thereof.

Without having the expertise to get into respective mathematical detail, I take it for granted here, that similarity by degree is used with algorithms for cluster analysis and more generally for pattern recognition.¹² This includes, but won't be restricted to, applications on a wide variety of technical devices, e.g. a plant recognition app on a smart phone.

There is a more traditional well known application of similarity by degree too, it's an industrial one and concerns rolling element bearings, as used e.g. in a car. For the performance (e.g. friction from rolling resistance) of such bearings, the precision of produced rolling elements (balls, rollers,...), wrt e.g. diameter deviations, has to be secured.

3.3 mentioning use cases - mostly from taxonomy

3.3.1 taxonomy in biology

Again, without being an expert in biology, rather obviously, the taxonomy framework in biology has changed significantly, since Whewell published his 'History of Scientific Ideas' in 1858. Already C.G. Hempel in his 'Fundamentals of Taxonomy' (1965) mentions a switch from the primarily morphological to the phylogenetic taxonomy paradigm, and welcomes it as a definite progress (Hempel [12], p. 147 f.), which it obviously is¹³. But, as any approach is accompanied by its own difficulties, there seem to exist paraphyletic and polyphyletic exceptions to the targeted monophyletic structures¹⁴. And it will be interesting, how much of the evolution theorist's apparent preference for monophyletic trees will in the long run turn out to be substantiated by computational phylogenetics.

Anyway, the involvement of similarity considerations in various ways in this field of research (e.g. in comparing aligned DNA sequences [similarity by degree, I guess], in selecting a most plausible phylogenetic branching pattern, if DNA sequences evidence is not already conclusive, ...) seems obvious. And, of course very important, concerning this use case, are first, the switch from mostly comparative similarity to mostly similarity by degree (enabled by a change of the

¹²reference here is to the respective articles "cluster analysis"[21] and "pattern recognition"[22] in wikipedia.

¹³perhaps a characteristic facet of the resulting situation is given by "traditional morphological analysis and analysis based on genetic data give us different phylogenetic trees ..." and a resulting question "[How] can we reach an equilibrium between classical and molecular classifications?" I'm indebted to Professor Emeritus Ahmed Thandar, marine biologist at the University of KwaZulu-Natal, for this light shedding remark (in personal communication). Any errors or inadequacies in referring to his remark are entirely mine

¹⁴reference is to articles "cladistics" [19], "computational phylogenetics" [20] etc. in wikipedia

object domain of inquiry to molecular facts), and secondly, the use of computational methods (algorithms) in determining and evaluating similarities.

3.3.2 in the humanities

And again, without being an humanities expert, there are some well known fields of study in the humanities, which could well be imagined their research objects being grouped by similarities, which in the general case will turn out to be non transitive similarities¹⁵. Some cases, coming to my mind immediately:

- comparative study of structure, growth, performance and decline of cities (in history and contemporary, synchronous or asynchronous)
- comparative study of formal and informal structure of organizations¹⁶
- on behalf of whatever enterprises or political groups, cluster analysis (using presumably similarity by degree) to select target groups
 - for advertising, e.g. from evaluating data warehoused customer data
 - for advertising else influencing, e.g. from evaluating user data of social media communities .

4 metaphysics use case in exempting Plato's theory of forms from Aristotle's criticism ?

4.1 disclaimer

Now, when recalling the historical origin of the seemingly endless debate 'on the relation of universals and particulars' (Russell's wording), viz., Aristotle's criticism of his picture of Plato's theory of forms in 'metaphysics' M9 as my example, and then try a defense of Plato's theory of forms by use of twentieth century first order logic, the anachronism is obvious, and anything I judge from this rather modern perspective may be criticized as inadequate or unjust to the historical players for this anachronistic procedere, and may as well be criticized for that text details may disallow my favored interpretation. My answer is, that my intention is not so much a historical, but a systematic one, and the simple reason, why then for heaven's sake I chose to select that historical origin for my case study is, that for getting an acceptable level of understanding of the problem, we have to some extent to realize and respect the use and intentions of the theory of forms in Plato's dialogues and only thence of Aristotle's picturing it. So I will try a balancing act between historical scrutiny to some extent and mapping part of the debate to the standard logic of our days.

¹⁵some attention to the use of types in the social sciences is given a in C.G. Hempel's 'Typological Methods in the Natural and the Social Sciences' [12] pp. 155-171

¹⁶reference is to Encyclopedia Britannica [8], [17]

4.2 the story

Let's now set the stage for the logical case study involving similarity, led by some scholarly work in the area, viz. Laura M. Castelli 'Universals, Particulars and Aristotle's Criticism of Plato's Forms'[6](2013) mainly for referencing Aristotle's view, and R. E. Allen 'Participation and Predication in Plato's Middle Dialogues'[1](1960) for a line of defense in favor of Plato's theory of forms/ideas. Allen's paper is also referred to by Castelli.

4.2.1 the attack

In part I 'The ontological status of Platonic Forms and the difficulties it poses' of her comprehensive paper Laura M. Castelli discusses as a main point the ἀπορία from metaphysics M9 which, according to Aristotle's coining of the case, the defenders of the theory of forms/ideas cannot avoid.

Aristotle claims, that the defenders of the theory of forms/ideas¹⁷ run into a difficulty close to a blatant contradiction¹⁸, in Aristotle's own words (in Ross translation) of [3] M9 1086a32-35: ... For they at the same time make the Ideas universal and again treat them as separable and as individuals. That this is not possible has been argued before¹⁹...

For interpretation and evaluation, attention is drawn by Castelli to an arsenal of further text passages, let's select only some:

A6, 987b1 : ... Socrates, however, was busying himself about ethical matters and ... seeking the universal (καθόλου) in these ethical matters, and fixed thought for the first time on definitions(ὀρισμῶν); ...

the passage continues

A6, 987 b4-b14: ... Plato accepted his teaching but held that the problem not applied to sensible things but to entities of another kind ... Things of this other sort, then he called Ideas, and sensible things, he said, were all named after these, and in virtue of a relation to these; for the many existed by participation (μέθεξις) in the Ideas that have the same name as they. Only the name 'participation' was new; for the Pythagoreans say that things exist by 'imitation' [of numbers²⁰] and Plato says they exist by participation, changing the name. But what the participation or the imitation(μιμήσις) could be they left an open question. ...

For the important passage on the role of universals for knowledge let's cite from Castelli's summarizing Aristotle's point of view:

"The problem is presented in Met., B4, 999a26-32 in the following terms: if there is nothing over and above particulars (τὰ δὲ καθ' ἕκαστα) and particulars are in(de)finite (ἄπειρα), there can be no ἐπιστήμη. Therefore there must be

¹⁷"... In his reports Aristotle uses the Greek equivalents of both 'Forms' and 'Ideas' (εἶδος and ἰδέα respectively), both appearing in the dialogues as well." Castelli [6], p. 139, note 1

¹⁸met. [3] M9, 1086a24-b14

¹⁹for the ontological difference between particulars and universals, besides others, some passages of Z13 are prominent ones,

Z13 1038b35-1039a1 (... no universal attribute is a substance ...)

1040a8-a9 (Nor is it possible to define any idea. For the idea is, as the supporters say, an individual and can exist apart ...)

this latter passage but question begging as relying on the questionable truth of B3, 999b26-29,(referred to by Castelli [6], p. 143, see below)

²⁰my exclusion

something which makes all things knowable in as much as it is one and the same (ἡ ἕν τι καὶ ταῦτόν), universal (ἡ καθόλου) or common to many things. But if this is the case, there must be something which is over and above particulars.” Castelli[6], p.143

4.2.2 the defense line

In a later section of the paper, '1.4. Paradigms, separation and the particularity of Forms', Castelli mentions and discusses a line of defense in favor of Plato's theory of forms based on the claim, that forms/ideas are not universals but paradigms²¹.

For details of a description of Plato's theory of forms/ideas which allows for this defense I'm going to cite from R. E. Allen's 'Participation and Predication in Plato's Middle Dialogues'[1][1960], especially from Allen's sections II (Plato's Theory of Predication), III(Imitation and Degrees of Reality), and V (Participation):

” II. Plato's Theory of Predication

Plato has no word for "predication." Rather he says that particulars are "called by the same name" (ὁμώνυμον) as their Form.

...

Each of the Forms exists, and the other things which come to have a share in them are named after them. The reason for naming particulars after Forms is that they have in them an immanent character defined by their Form: ...

...

Not only is the Form itself always entitled to its own name, but also what is not the Form, but always has, when it exists, its immanent character (μορφή).

...

These passages imply that "F" is a name, a name whose prime designate is a Form: "F" names the F. But this name is also applied, through what we may call derivative designation, to particulars, which are named after the Form in much the way that a boy may be named after his father. The reason for this, the justification for derivative designation, is that particulars have in them the immanent character defined by their Form; or, to put the matter in a slightly different way, they are named after the Form because of their peculiarly intimate relation to it-they depend upon it for their character and their existence.

We have, then, a theory of predication without predicates.” [1], pp.149-150

” III. Imitation and Degrees of Reality

The theory of Forms involves two fundamental doctrines: (a) that the relation between particulars and Forms is that of imitation, of copy to original, and (b) that Forms and particulars differ in degree of reality.”

”V. Participation

...

The particular objects of sense are unified by a One which stands on a different level of reality from theirs; their community of character is to be explained

²¹the tradition of this line of defense, we learn, started with work of P.T. Geach in 1956, followed by work of R. E. Allen and other scholars, see [6], pp. 152ff

by the introduction of Forms. Unity and diversity are reconciled if we posit the existence of two domains, Being and Becoming, a world of particulars, of things unified, and a world of Forms, their unity. To understand the One and the Many, we must understand that the One is over the Many. ..." [1], p.160

The pros and cons for this line of defense, considered by scholars in the field, pertain to items of rather equal antiquity like 'the third man' (understood from Plato's Parmenides dialogue), or by a related topic, 'self-predication', a rather dubious 'logical' characterization, but interpolated from some of Plato's dialogues, and seemingly current in the field. Allen, who contests (imop rightly) the alleged 'self-reference', explains the provenance of the item:

"... Plato obviously accepts the following thesis: some (perhaps all) entities which may be designated by a phrase of the form "the F Itself," or any synonyms thereof, may be called F. So the Beautiful Itself will be beautiful, the Just Itself just, Equality equal. ..." and Allen points to respective text passages in 'Protagoras', 'Phaedo' and others [[1], section I. Self-Predication, p.148]

At this stage I leave the scholarly discussion and try a free logical construct, inspired by the scenario, developed so far.

4.3 the defense constructed as a similarity use case

The following logical construct shall picture the given scenario to some extent, and show release from the impression of an aporetic epistemic situation.²²

4.3.1 target of construction

From Allen's exposition one takes easily, that in Plato's theory of forms sensible particulars are mapped by their 'immanent character' to their form.

We make use of the assumption, that this mapping may be well considered to be a mapping or function in the mathematical sense (viz., providing a uniquely determined function value for each of it's arguments). This - in context - amounts to constructing a mapping or function, the set of arguments of this function - it's domain - is a union of (not necessarily mutually disjoint) subsets of concrete (e.g. sensible) particulars²³. The value of the mapping for a special argument (i.e. for a special concrete particulars from this union set) is the

²²There is already a fairly formal account of the respective logical structure of Plato's view in P.T. Geach's article of 1956, that started the 'paradigm defense line' discussion, viz.

"I shall now state in an abstract logical way what seem to me to be Plato's implicit assumptions in the TMA (and the presuppositions of the Broadman-Izzard discussion).

(i) There is a set consisting just of the many Fs that are not Forms.
 (2) If x is a Form by which y is made to be an F, then y is not a Form by which x is made to be an F.

(3) If A is a set of several Fs, and x is an F not belonging to A, then there is a set of Fs containing just the members of A together with x. (I shall call this set "A plus x.")

(4a) Some F is a Form by which all other Fs are made to be Fs.
 (4b) Any set consisting of several Fs are all of them made to be Fs by a Form that is itself an F."
 ([9], pp. 77 f).

I do not follow Geach's further exposition in that paper. The problem - in my view - is not, that these four theses are taken to reformulate Plato's view; in the opposite, I do agree more or less with each of them. My point of disagreement is rather with what in the sequel is assumed by Geach to follow from them, especially wrt his reading of the 'Parmenides'. But this would need a discussion of its own, and is not my focus here.

²³while, with regard to Plato's dialogues, we might perhaps be inclined to restrict discussion to sensible particulars, the general case of particulars covered here is but somewhat more broadly

form/idea, the respective particular is said to participate in by implementing its character. The construct is intended to sketch such a mapping and to some extent show it's trade off.

4.3.2 construction

1. The set of concrete particulars be pre-structured by (epistemically relevant, but hitherto not sufficiently analyzed) similarities in sample sets (these not necessarily mutually disjoint).

2. By analyzing prominent examples of such epistemically relevant similarity samples (e.g. of natural kinds) with regard to their properties, and by some steps of abstraction (by propositional invariance, and, in case of gradable properties of the analyzed examples, by optionally postulating a maximal value of that property), in case of successful analysis we end up with a concept cluster abstracted from the prominent examples²⁴. This concept cluster is taken to characterize

an abstract particular, uniquely determined by this concept cluster²⁵

that concept cluster being it's definite description

3. Hence, there is for any such successful analysis of selected examples, a unique abstract particular, being the only particular implementing this concept cluster in full. We use the big latin letter '**F**' to refer to the concept cluster, which is a complex predicate [picturing Allen's immanent character($\mu\omicron\rho\varphi\eta$)], and the 0-ary function symbol '**i**⁰' to refer to the abstract particular, and write **F**(**i**⁰) [or, omitting brackets: **F****i**⁰], to refer to the abstract particular's implementing this concept cluster in full,

this usage of the 0-ary function symbol **i**⁰ presupposing

$\forall \mathbf{y} [\mathbf{F}\mathbf{y} \wedge \wedge_{\mathbf{x}} (\mathbf{F}\mathbf{x} \rightarrow \mathbf{x} = \mathbf{y})]$ [existence and uniqueness, secured by **F**]

4. Now, having available the abstract particular **i**⁰, our formal analogon of a Platonic form/idea, we have to spell out the use of this abstract particular, uniquely identified by it's cluster concept, in giving a formal picture of the theory of forms:

construed: concrete (maybe or not sensible) objects, as e.g. a city, an organization (from the humanities use cases mentioned in section (3)) are concrete particulars, squares or triangles in pure geometry abstract particulars, drawn in the sand squares or triangles are concrete sensible particulars. "concrete" is here used in a usual way as opposed to "abstract"

²⁴introducing a 'concept cluster' as a means of logical reconstruction is not new in the context. E.g. Castelli, in another paper [5] makes a try, when tracing ... the problem of the universality or individuality of forms ... in Aristotle. And in that paper, she is critical on this kind of construction, but I hope to show here, that respective difficulties can be smoothed over. Of course, my construction differs from theirs in some points

²⁵this uniqueness may be viewed as a postulate

5. the concrete particulars from the respective similarity sample(s) are said to implement the concept cluster \mathbf{F} to a certain degree, if they are similar wrt \mathbf{F} to the abstract particular \mathbf{i}^0 in that degree. Alternatively they are said to participate in \mathbf{i}^0 . The degree of similarity varies, but it's never complete similarity (this is guaranteed by the abstraction process determining the concept cluster \mathbf{F}), and it's never non similarity, else they couldn't be said to participate.

6. Certain knowledge is possible only wrt \mathbf{i}^0 as implementing \mathbf{F} in full, i.e., possible only wrt properties of the abstract particular \mathbf{i}^0 . Knowledge of concrete particulars, in some or other way only deficiently implementing the concept cluster \mathbf{F} , is only indirect, is given only in so far, as they participate in \mathbf{i}^0 . This latter consequence of Plato's theory of forms, re-pictured here, is not so uncommon or implausible as one might expect; to take a well known simple example from classical mechanics: in a strict sense there are no (physically real) inertial systems, but of course the Galilei/Newton law of inertia, defining this concept.

7. The alleged difficulty of 'self predication' amounts, if one accepts the construct described so far, to nothing than a misunderstanding of logical structure. The examples from the dialogues, Allen lists in his exposition, may easily be understood as follows:

read being an \mathbf{F} as meaning being to some degree similar wrt \mathbf{F} to \mathbf{i}^0

And this applies to both, the abstract particular \mathbf{i}^0 as well as to any concrete (e.g. sensible) particular implementing \mathbf{F} . Of course, for \mathbf{i}^0 the unique abstract particular having the character \mathbf{F} in full, its similarity turns out to be identity (thus similarity degree $\mathbf{d} = \mathbf{1}$ in the similarity model, recalled in the beginning of this paper). For concrete particulars, implementing \mathbf{F} only to a certain degree, the similarity is less, but nonetheless similarity (thus in that model would be pictured by a degree \mathbf{d} with $\cos 60^\circ = \mathbf{0}, \mathbf{5} \leq \mathbf{d} < \mathbf{1} = \cos 0^\circ$).

And hence one can (with reference again to the geometrical similarity model) roughly formalize the above given read of ' \mathbf{x} being an \mathbf{F} ' [for short $\mathbf{bF}(\mathbf{x})$] as

$$\mathbf{bF}(\mathbf{x}) \Leftrightarrow \exists \mathbf{d} (\delta_{\text{SIM}_{\text{wrt } \mathbf{F}} \text{ to } \mathbf{i}^0}(\mathbf{x}) = \mathbf{d} \wedge \cos 60^\circ = \mathbf{0}, \mathbf{5} \leq \mathbf{d} \leq \mathbf{1} = \cos 0^\circ)$$

where δ is a function, taken to map the similarity wrt \mathbf{F} to the geometric similarity model. The important thing wrt ruling out the allegation of 'self predication', is the existential quantification [... being an \mathbf{F} ... means ... there is an similarity degree \mathbf{d} , such that ...]

8. Now, is there something to be said with respect to transitivity or non transitivity of 'similarity wrt \mathbf{F} ' ?

OK, in the first place this similarity is given between the abstract particular \mathbf{i}^0 and any concrete particular \mathbf{x} , implementing \mathbf{F} in some degree. This scenario is represented by the complex unary predicate $\mathbf{bF}(\mathbf{x})$. And the trivial equivalence relation given with $\mathbf{bF}(\mathbf{x})$, viz. $\mathbf{bF}(\mathbf{x}) \leftrightarrow \mathbf{bF}(\mathbf{y})$, is of course transitive.

But what about 'similarity wrt \mathbf{F} ' between concrete particulars, say \mathbf{x} and \mathbf{y} ? As \mathbf{F} is typically a cluster concept, it may well be in limit cases that the

intersection of the respective properties from \mathbf{F} , the ones shared by \mathbf{x} and the ones shared by \mathbf{y} is even empty. Thus, with respect to a domain constituted by the abstract particular \mathbf{i}^0 and all related concrete particulars \mathbf{x} there is no good reason to expect the relation 'similarity wrt \mathbf{F} ' in the general case to be transitive.

9. At this point it might be helpful to recall, what formal explicata have been correlated to which traditional terms in this (re-)construction so far:

cluster concept \mathbf{F}

correlates to a Platonic form's 'immanent character ($\mu\omicron\rho\phi\eta$)' (Allen)

abstract particular \mathbf{i}^0 , uniquely determined by \mathbf{F} ,

correlates to (is our formal analogon of) a Platonic form/idea ($\epsilon\tilde{\iota}\delta\omicron\varsigma$ / $\iota\delta\acute{\epsilon}\alpha$)

similarity degree $\delta_{\mathbf{SIM}_{\text{wrt } \mathbf{F}} \text{ to } \mathbf{i}^0}$

correlates to 'participation/imitation ($\mu\acute{\epsilon}\theta\epsilon\zeta\iota\varsigma$ / $\mu\iota\mu\eta\sigma\iota\varsigma$)'

the complex unary predicate \mathbf{bF}

correlates to 'being an \mathbf{F} ',

which in turn correlates to Aristotle's universal ($\kappa\alpha\theta\acute{\omicron}\lambda\omicron\upsilon$)

10. The reconstruction of the theory of forms, proposed here, does allow for definition, does allow for certainty of knowledge, but both only for the abstract particular \mathbf{i}^0 , not for the participating concrete particulars; and thus prima facie looks totally different from an Aristotelian concept of certain science, based on certain first principles, formulated in general statements, employing universal concepts (with supposedly definite extension).

5 off the record

degrees of reality ?

My (re-)construction considers rather explicitly Allen's first 'fundamental doctrine' involved by the theory of forms, viz. "(a) that the relation between particulars and Forms is that of imitation, of copy to original" [1], p. 152, but seems to neglect his second one, viz. "(b) that Forms and particulars differ in degree of reality"(ibid.).

Well, I did not really neglect this point, for I commented on degrees of knowability (... certain knowledge only of the forms ...). And as I hold it a perhaps problematic short circuit, to attribute to Plato identification of degrees of knowability with degrees of reality, I stopped (only) short of that²⁶.

in favor of the Meno

My confidence in the presented reconstruction is based to considerable extent on Plato's presentation of the geometry example in the Meno (82b9-85b7),

²⁶the text passages, Allen alludes to in favor of 'degrees of reality' [[1] p.155 fn 18, fn 19], aren't decisive wrt this difference, e.g. Phaedo [15] 74d5-7 in my perception may very well be interpreted as referring to 'degrees of knowability', not necessarily implying reference to 'degrees of reality'

which is usually considered as prominent example, where Plato argues for the innateness thesis. May be, but much more important, in my perception, is the accentuation of the role, geometry plays as the standard example for plausibility, applicability and validity of the theory of forms/ideas as a theory of knowledge²⁷. The moment of Meno's slave boy realizing the correct construction for doubling the square is the moment of acquiring knowledge [but only, because he had no education in geometry before] by successfully perceiving an aspect of the form of a square²⁸.

Whewell emerging from the history of the philosophy of science

My first perception of the Whewell-Mill debate was a quotation of two text passages in 'A Wittgenstein Workbook' by Christopher Coope, Peter Geach, Timothy Potts and Roger White (Blackwell, Oxford 1970). The text passages were assigned explicitly as a supporting material for the discussion of the later Wittgenstein's talk of language games as related by family resemblances in "Philosophical Investigations", obviously in order to give Wittgenstein's refusal to define his term 'language game' some plausibility. For me²⁹ not so much Wittgenstein's usage of the term 'Sprachspiel(language game)' was puzzling, but the logical question raised by the debate between Whewell and Mill. This latter question seemed and seems to me more important in its own right.

6 summing up

I drew attention to the practical and to the epistemic role of (in general) non transitive similarity relations by citing some examples, sure, lots of them could be added, e.g. from the band with of cognitive sciences, including learning and/or behavioral analysis. And I was asking for more due consideration of them³⁰ within contemporary epistemology.

With reference to a well known example, wavering throughout the history of philosophy, I constructed an hopefully significant use case. But what is it, that allows for such an use case of an item of ancient metaphysics, which has not already been available at the time of Aristotle ? Or even to Late Middle Age philosophers, ruminating (forgive me) the problem [in the way Aristotle presented it] again and again ? The answer, that it is the availability of a

²⁷Socrates climbing down in Meno[14]86b4-10 "Meno: What you say commends itself to me, Socrates, I know not how. Socrates: And so it does to me, Meno. Most of the points I have made in support of my argument are not such as I can confidently assert; but that the belief in the duty of inquiring after what we do not know will make us better and braver and less helpless than the notion that there is not even a possibility of discovering what we do not know ...", shows that the core interest is the 'that' of knowledge acquisition by perceiving a form, not some story about the alleged causal chain (innateness etc.) for this 'that'.

²⁸one should not be distracted from this perception by e.g. the report of Aristotle in [3] 987b14-b18, viz. that Plato allegedly assigned an only intermediate ontological status to the objects of mathematics between sensible things and forms, which view unfortunately had a late echo e.g. in Proclus' introduction to his Euclid commentary

²⁹then the German translator of this nice booklet, translation appeared 1972 at Suhrkamp/Ffm. as a supplementary ("Beiheft 2") to their Wittgenstein-Edition

³⁰more recently there has been *some* awareness of the role of non transitivity, viz. with some scholars wrt the Sorites discussion see e.g. "...our similarity-based semantics for first-order logic rests on the idea that vagueness is tied in an essential way to non-transitivity, whether of indifference or indiscriminability. In this, the framework agrees in particular with one of the central hypotheses of Williamson's epistemic theory of vagueness ..." [7] pp.383 f.

comfortable formal tool, viz. 1st order logic, including logic of relations and functions, is only part of the story. The perhaps as well important other part of the story is, that in the age of industry and information technology (already referred to) we are in a technologically superior position to realize not only the ubiquity, but the objectivity of non transitive similarity (if we think e.g. of the impact of the degree of similarity of the rolling elements (e.g. balls) on the performance of a rolling bearing)³¹.

The objectivity of non transitive similarity is expressed in the view that, even with the best available knowledge, the most precise available measurements, cases of non transitive similarity will not only remain to exist, but often will be reproducible and/or predictable. Hence, the suggestion, that arguments from non transitive similarity show principally a deficiency wrt knowledge [as occasionally insinuated e.g. in the Sorites discussion], seems, from this point of view, to be systematically misleading and in the end untenable.

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³¹while of course already in Greece at round about Socrates' time, replica techniques of different kinds will have been available, e.g. bronze cast, and coins made of gold, silver, and perhaps already of bronze, were then in use; so they had their own experience of technically controlled similarity.

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