

Misleading Interaction and Curvilinear Terms

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This article examines the relationships between interaction (product) terms and curvilinear (quadratic) terms in regression models in which the independent variables are correlated. The author uses 2 substantive examples to demonstrate the following outcomes: (a) If the appropriate quadratic terms are not added to the estimated model, then the observed interaction may indicate a synergistic (offsetting) relationship between the independent variables, whereas the true relationship is, in fact, offsetting (synergistic). (b) If the appropriate product terms are not added to the equation, then the estimated model may indicate concave (convex) relationships between the independent variables and the dependent variable, whereas the true relationship is, in fact, convex (concave). (c) If the appropriate product and quadratic terms are not examined simultaneously, then the observed interactive or curvilinear relationships may be nonsignificant when such relationships exist. The implications of these results for the examination of interaction and quadratic effects in multiple regression analysis are discussed.

Hypotheses about interaction effects between continuous variables are frequently examined in psychological research using multiple regression analysis (e.g., Aiken & West, 1991; Jaccard, Turrisi, & Wan, 1990). However, despite the plethora of research concerning these hypotheses, the appropriate methods to test them remain a subject of debate. One issue that has received much attention in the recent literature is the relationship between interaction and curvilinear effects when there is a high multicollinearity between independent variables (e.g., Busemeyer & Jones, 1983; Lubinski & Humphreys, 1990; Shepperd, 1991; MacCallum & Marr, 1995). Using two substantive examples, I investigate the outcome of omitting one of these two nonlinear effects when the true model includes both of them and multicollinearity is high, and I discuss the implications of these examples for the process of fitting nonlinear models to empirical data.

An interaction between two independent variables is said to occur when the impact of one independent variable on the dependent variable depends on the

level of another independent variable. When there are two independent variables, X and Z , and one dependent variable, Y , interaction is usually conceptualized in terms of the effect of the product XZ on Y . Note that an interaction, as defined above, need not necessarily be represented by the product of the independent variables. However, for the sake of simplicity, in the current article I assume that the true interactive relationship between two variables is indeed associated with their product.

The existence of a “true” interaction is usually examined by introducing a product term into a linear model and observing the value of β_3 in the following regression equation:

$$Y = \beta_0 + \beta_1X + \beta_2Z + \beta_3XZ. \quad (1)$$

A significant value of β_3 indicates the existence of an interaction. A positive value of β_3 indicates that the true relationship between Y and XZ is positive (a positive interaction), whereas a negative value of β_3 indicates that the true relationship between Y and XZ is negative (a negative interaction). When X and Z are continuous and positively related to Y , a negative interaction suggests an underlying offsetting relationship between the two independent variables, whereas a positive interaction suggests an underlying synergistic relationship between them. An *offsetting relationship* implies that the relationship between X and Y becomes more positive as the value of Z decreases,

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whereas a *synergistic relationship* implies that the relationship between X and Y becomes more positive as the value of Z increases (obviously, this statement is symmetric with regard to X and Z). Note that the concepts of offsetting and synergistic relationships are meaningful only when the relationship between the dependent variable and each of the independent variables is conditionally monotone (does not involve a cross-over interaction); that is, when an increase in one independent variable results in an increase in the dependent variable for every value of the other independent variable. This relationship characterizes many psychological phenomena (Dawes & Corrigan, 1974). For example, no matter how students score on other tests, their mathematical achievement is likely to be higher the higher they score on a mathematical reasoning test or on a spatial reasoning test. No matter how high the education of the father, the educational expectations of the child are likely to increase with the education of the mother.

Curvilinearity is said to occur when the functional relationship between the dependent and the independent variables is negatively accelerated (concave) or positively accelerated (convex). Such relationships are usually examined by adding quadratic terms to a linear model and observing the value of their coefficients:

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 X^2 + \beta_4 Z^2. \quad (2)$$

A significantly negative value of β_3 indicates a concave relationship between X and Y, whereas a significantly positive value indicates a convex relationship (similarly, a significantly negative value of β_4 indicates a concave relationship between Z and Y, whereas a significantly positive value indicates a convex relationship). Note that a curvilinear relationship as defined above need not necessarily be quadratic. However, for the sake of simplicity, in the current article I assume that a true curvilinear relationship is indeed quadratic.

Finally, note also that the above discussion does not distinguish between observed measures and latent variables. The distinction between the two is not necessary for examining the basic issues associated with the effect of model misspecification. However, because these issues are important in applied research, they are discussed in the *Discussion* section. In particular, I discuss the effect of the level of measurement and the reliability of the measurement on the outcome of model misspecification.

Misleading Interaction Terms

Though the study of interactions among variables using product terms is common enough in the social sciences, a number of problems are associated with this method of analysis. In particular, when there is multicollinearity between the independent variables, the observed interaction may be spurious; that is, the coefficient of the product term in the regression may be significant even when there is no true interaction. The reason for this is that when the correlation between X and Z increases so does the correlation between XZ and X^2 , which results in an overlap between the variance explained by XZ and the variance explained by X^2 . Busemeyer and Jones (1983) presented an analytic discussion of such "spurious" interaction, and Lubinski and Humphreys (1990) provided a relevant substantive example.¹

Even more troubling than spurious interactions are misleading interaction terms. A *misleading interaction term* occurs when the observed interaction term in Equation 1 is positive, while the true interaction is negative, or when the observed interaction term is negative, while the true interaction is positive (a negative suppression situation; see Darlington, 1968; Tzelgov & Henik, 1991). This may happen when, concur-

¹ In the example of Lubinski and Humphreys (1990), advanced mathematics measure (labelled C) was regressed on mathematical reasoning (labelled M) and on spatial reasoning (labelled S) in three consecutive stages. In the first stage, C was regressed on M and S; in the second stage, C was regressed on M, S, and MS; and in the third stage, M and S were forced into the regression, and M^2 , S^2 , and MS were then entered simultaneously in a stepwise procedure. The results of the first stage indicated a strong effect for M but no effect for S. The results of the second stage indicated a strong effect for M and MS but no effect for S. The results of the third stage indicated an effect for M and M^2 , and no other effects. On the basis of these results, Lubinski and Humphreys concluded that the effect of MS was spurious. However, this example is problematic because the second stage is itself "spurious": It could be concluded a priori (even without estimating the models of the third stage) that the effect of MS in the second stage is spurious. The reason for this is that such an effect could be associated only with a crossover interaction, in which there are levels of M for which an increase in S leads to a decrease in C. That is, given a conditionally monotone relationship between M, S, and C, a nonsignificant main effect of S in the first stage implies that the interaction cannot be significant. Thus, any observed interaction must be spurious.

Table 1
The Effect of the True Interaction Coefficient (b) and the Correlation Between the Predictors (r) on the Probability of Obtaining a Significant Misleading Interaction in a Misspecified Model

b	r				
	0.0	0.1	0.3	0.5	0.7
0.1	36/2 (.01)	0/73 (.01)	0/100 (.01)	0/100 (.01)	0/100 (.00)
0.3	74/0 (.08)	3/34 (.08)	0/100 (.07)	0/100 (.04)	0/100 (.02)
0.5	97/0 (.20)	31/1 (.20)	0/100 (.16)	0/100 (.10)	0/100 (.04)
0.8	100/0 (.39)	90/0 (.38)	0/89 (.33)	0/100 (.22)	0/100 (.10)
1.0	100/0 (.50)	100/0 (.49)	3/34 (.43)	0/100 (.30)	0/100 (.15)
1.2	100/0 (.58)	100/0 (.58)	32/3 (.52)	0/100 (.39)	0/100 (.20)
1.5	100/0 (.69)	100/0 (.69)	100/0 (.63)	0/39 (.50)	0/100 (.27)
2.0	100/0 (.80)	100/0 (.79)	100/0 (.75)	100/0 (.64)	100/0 (.40)

Note. The model that was estimated was $Y = \beta_0 + \beta_1X + \beta_2Z + \beta_3XZ$, whereas the true model was $Y = X + Z + X^2 + Z^2 - bXZ + E$. Five hundred simulations (each with $n = 1,000$) were conducted for each $b-r$ pair. For each of the 48 pairs, the number to the left of the slash represents the percentage of simulations in which the sign of β_3 was correct (significantly negative, $p < .01$) and the number to the right of the slash represents the percentage of simulations for which the sign of β_3 was incorrect (significantly positive). The numbers in parentheses are the mean squared partial correlations of the product term, controlling for the other terms of the true model.

rent with a true interaction, there exists a true curvilinear relationship between dependent and independent variables, as well as a multicollinearity between the independent variables, and the analyst misspecifies the model by not including curvilinear terms. In particular, an observed negative interaction term can occur when, concurrent with a true positive interaction, there is also a true concave trend; and an observed positive interaction term will occur when, concurrent with a true negative interaction, there is also a true convex trend. In this case, when the multicollinearity between X and Z increases, so does the correlation between XZ and the quadratic terms. This results in an increasing amount of the variance being shared by the product term and the quadratic terms.

As an example, consider the following simulation in which $n = 1,000$; X , Z , and E (an error term) are normally distributed with a mean of 0 and standard deviation of 1, the correlation between X and Z is .7, and the true value of Y is given by $Y = X + Z + X^2 + Z^2 - XZ + E$ (that is, the true interaction is negative). In each of 500 such simulations, the regression $Y = \beta_0 + \beta_1X + \beta_2Z + \beta_3XZ$ yielded a significantly positive coefficient for XZ .² In other words, not including the appropriate quadratic terms in the regression would have led to the conclusion that the relationship between X and Z is synergistic, when, in fact, the true relationship is offsetting.³ Table 1 shows the effect of the true interaction coefficient and the correlation between the predictors on the probability of

obtaining a misleading interaction in a misspecified model. It demonstrates that this probability is high over a large range of correlations and a large range of differences between the effect size of the product and quadratic terms.

Misleading Curvilinear Terms

The multicollinearity between XZ and the quadratic terms may also lead to concluding the existence of misleading curvilinear terms. *Misleading curvilinear terms* may occur when the observed curvilinear relationships between the dependent variables and the independent variable are concave, whereas the true relationships are convex; or when these relationships are convex, whereas the true relationships are concave. This may happen when, concurrent with true curvilinear relationships, there is a (true) interaction, as well as a multicollinearity, between the two independent variables, and the analyst misspecifies the model by not including the product term. As a result, when

² The significance levels in the simulations are .01.

³ The role of multicollinearity in producing these results becomes apparent if one notes that when the correlation between X and Z is 0, XZ is orthogonal both to X^2 and Z^2 . Thus, in this case, there is no shared variance between the multiplicative term and the curvilinear terms. On the other hand, when the correlation between X and Z approaches 1, the unshared variance is very small.

the multicollinearity between X and Z increases, more variance is shared by the product term and the quadratic terms.

As an example, consider a simulation similar to the one discussed above (i.e., the same distributions and correlation of the independent variables) except that the true value of Y is given by $Y = X + Z + .7X^2 + .7Z^2 - 2XZ + E$. In each of 500 such simulations, the regression $Y = \beta_0 + \beta_1X + \beta_2Z + \beta_3X^2 + \beta_4Z^2$ yielded significantly negative coefficients for X^2 and Z^2 . That is, not including the appropriate product term in the regression would have led to the conclusion that the curvilinear relationships between the dependent variables and the independent variable are concave, whereas the true relationships are convex. As Table 2 shows, this effect occurs over a large range of correlations and a large range of differences between the effect size of the product and quadratic terms.

Reciprocal Suppression Effects

There may be instances in which both true curvilinear and true interactive relationships occur, but neither the curvilinear terms nor the interaction terms are significant when estimated alone (a reciprocal suppression situation, as in Conger, 1974; Tzelgov & Henik, 1991).

As an example, consider yet another simulation, with the same distributions and correlation of the independent variables as the previous simulations, in

which the true value of Y is given by $Y = X + Z + X^2 + Z^2 - 2XZ + 6E$. In only 3% of 500 such simulations did the regression $Y = \beta_0 + \beta_1X + \beta_2Z + \beta_3XZ$ yield a significant (negative) coefficient for XZ , and in only 1% of these simulations did the regression $Y = \beta_0 + \beta_1X + \beta_2Z + \beta_3X^2 + \beta_4Z^2$ yield significant (positive) coefficients for X^2 and Z^2 . On the other hand, the regression $Y = \beta_0 + \beta_1X + \beta_2Z + \beta_3X^2 + \beta_4Z^2 + \beta_5XZ$ yielded a significant negative coefficient of XZ in 96% of these simulations, and a significant positive coefficient of X^2 and Z^2 in 87% of these simulations.

Organization of the Article

The article focuses on issues associated with misleading interaction and curvilinear terms, using information derived from two widely used databases in the context of two substantive issues: the impact of parents' education on their children's academic expectations and the integration of information in clinical judgments. The first section of the article demonstrates a misleading interaction term in a simple case in which there are only two independent variables. The second section demonstrates misleading curvilinear terms and reciprocal suppression in a more complex case, involving 11 independent variables. Finally, in the last section I discuss the implications of these demonstrations for the examination of interaction and quadratic effects in multiple regression analysis.

Table 2
The Effect of the True Quadratic Coefficient (b) and the Correlation Between the Predictors (r) on the Probability of Obtaining a Significant Misleading Curvilinear Terms in a Misspecified Model

b	r				
	0.0	0.1	0.3	0.5	0.7
0.1	39/2 (.02)	1/38 (.02)	0/97 (.02)	0/100 (.01)	0/100 (.01)
0.3	94/0 (.15)	42/1 (.15)	0/94 (.13)	0/100 (.09)	0/100 (.4)
0.5	100/0 (.33)	96/0 (.33)	4/24 (.29)	0/100 (.22)	0/100 (.11)
0.7	100/0 (.49)	100/0 (.49)	70/0 (.45)	0/60 (.35)	0/100 (.20)
0.9	100/0 (.62)	100/0 (.61)	100/0 (.56)	53/0 (.48)	0/14 (.29)
1.5	100/0 (.81)	100/0 (.81)	100/0 (.78)	100/0 (.72)	100/0 (.53)

Note. The model that was estimated was $Y = \beta_0 + \beta_1X + \beta_2Z + \beta_3X^2 + \beta_4Z^2$, while the true model was $Y = X + Z + bX^2 + bZ^2 - 2XZ + E$. Five hundred simulations (each with $n = 1,000$) were conducted for each $b-r$ pair. For each of the 48 pairs, the number to the left of the slash represents the percentage of simulations in which the sign of β_3 was correct (significantly positive, $p < .01$) and the number to the right of the slash represents the percentage of simulations for which the sign of β_3 was incorrect (significantly negative). The numbers in parentheses are the mean squared partial correlations of a quadratic term, controlling for the other terms in the true model.

Table 3
Intercorrelations Among the NLSY Variables

Variable	1	2	3	4	5
1. Educational expectations (EE)	—				
2. Mother's education (ME)	.38	—			
3. Father's education (FE)	.40	.67	—		
4. ME ²	-.05	-.46	-.28	—	
5. FE ²	.02	-.24	-.24	.53	—
6. FE × EE	-.03	-.37	-.25	.83	.76

Note. NLSY = National Longitudinal Survey of Youth.

Parents' Education and Children's Educational Expectations: The National Longitudinal Survey of Youth (NLSY) Data

Data

The data were taken from the NLSY (Center for Human Resource Research, 1995), conducted with a probability sample of 12,686 persons (with an oversampling of African Americans, Hispanics, and economically disadvantaged Whites) born between 1957 and 1964. Three variables from the 1979 survey were used in the current analysis: (a) the educational expectations of the youths, expressed in terms of the number of years they expect to complete; (b) the highest grade achieved by the father; and (c) the highest grade achieved by the mother. In the analysis, I used all the 7,748 children who were living with both parents at the age of 14 (meaning that both parents were likely to be present at home at about the time the educational expectations were measured) and whose father's education, mother's education, and educational expectations were available. All independent variables were centered around their mean. Table 3 presents the intercorrelations among the variables and their quadratic and product terms.

Theory

There is much evidence suggesting that judgments are highly interactive: The impact of an input variable on judgment may depend on its rank vis-à-vis the other variables which serve as input for the judgment (Birnbaum & Stenger, 1981; Ganzach, 1993, 1994; Weber, 1994). Quite often, this tendency toward interactivity in judgment is associated with optimism. For example, in predicting academic success of fellow students, undergraduates tend to assign a larger weight to the more positive information (Ganzach & Krantz, 1991). Thus, the theory underlying the current analysis is that the education of the parent with the

higher level of schooling exerts more influence on the child's educational expectations than the education of the parent with less schooling. In other words, the theory predicts an offsetting relationship between the education of the father and the education of the mother. When the level of education of one parent is high, the educational expectations of the child will be relatively high, even if the level of education of the other parent is low.

Analysis

Our theory suggests a negative interaction between father's education and mother's education. Nonetheless, when regressing educational expectations on father's education, mother's education and a product term, the coefficient of the interaction is significantly positive,⁴ which—contrary to the theory—may suggest a synergistic relationship. The results of this regression are given in part 2 of Table 4 (part 1 of this table presents the details of the linear regression).

However, when the quadratic terms of father's education and mother's education are added to the regression, the expected interactive relationship between parents' education is revealed. The coefficient of the product term is significantly negative, suggesting an offsetting relationship between father's education and mother's education. The details of this regression are given in part 3 of Table 4.

The results also indicate a convex (positively accelerated) relationship between mother's education and educational expectations, as well as a convex relationship between father's education and educational expectations: Both quadratic coefficients are positive, and both are significantly different from 0. The reason for this pattern is that for less than 12 years of edu-

⁴ The significance level in the empirical examples are .0001.

Table 4
Regression Models of Educational Expectations

Terms	Estimated regression model	R ²
Linear	EE = 14.006 + .148 * ME + .159 * FE (.024) (.010) (.008)	.183
Linear + interaction	EE = 13.866 + .185 * ME + .159 * FE + .017 * ME * FE (.027) (.011) (.008) (.001)	.197
Linear + interaction + quadratic	EE = 13.708 + .191 * ME + .167 * FE - .012 * ME * FE + .014 * FE ² + .018 * ME ² (.031) (.011) (.008) (.003) (.002) (.002)	.208

Note. Numbers in parentheses are standard errors of the estimate. EE = educational expectations; FE = father's education; ME = mother's education.

cation, the relationship between parents' education and the child's educational expectations is only slightly positive, whereas for more than 12 years it is much more positive. This differential relationship between parents' education and educational expectations may be associated with the fact that below a normative level of 12 years of education, a change in parents' education is relatively less indicative of their attitudes toward education than above this threshold level, and therefore is more predictive of the child's educational expectations.

Finally, to illustrate the joint effect of both the curvilinear and the interactive relationships of parents' education, Figure 1 displays a plot of mean educa-

tional expectations against father's education, separately for high levels of mother's education (12 or more years) and low levels of mother's education (11 years or less). (Only levels of father's education for which there were enough available observations for each of the two levels of mother's education appear in the graph.) The convexity of the curves demonstrates the curvilinear relationship for both levels of mother's education; the increased distance between the curves when the value of mother's education decreases demonstrates the offsetting relationship between father's education and mother's education. For example, when father's education is low, the difference between the educational expectations of the children of mothers with high and low education is about .9 years, whereas when father's education is high, this difference is only .4 years.

Clinical Judgment: Meehl's Data

Data

The data were collected by Paul Meehl in the 1950s. They included judgments of 861 patients, diagnosed as being either neurotic or psychotic, on the basis of their Minnesota Multiphasic Personality Inventory (MMPI) profiles (their scores on the 11 scales of the MMPI). The judgments were made on an 11-step forced normal distribution from 1 (least psychotic) to 11 (most psychotic). Note that because the patients were diagnosed either as psychotics or as neurotics, low rating on this scale implies a judgment of high likelihood of neurosis. The evaluations were obtained from 13 clinical psychologists and 16 clinical psychology trainees (see Meehl, 1959, for a detailed description of the data).

One aspect of the stimuli that is particularly important to the current study is that the MMPI scales of the 861 profiles have a clear dimensional organization. One dimension is associated with the four neurotic scales of the MMPI; another with five psychotic

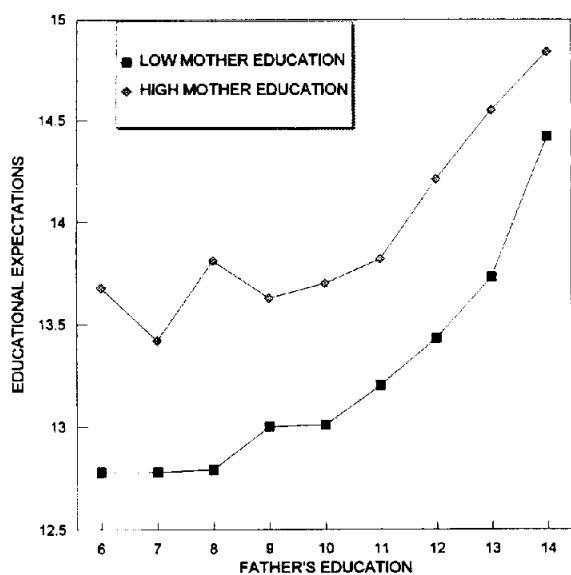


Figure 1. Mean educational expectations as a function of father's education, separately for high levels of mother's education (12 or more years) and low levels of mother's education (11 years or less). (Only levels of father's education for which there were enough available observations for each of the two levels of mother's education appear in the graph.)

scales; and the third dimension is associated with two scales that identify defensiveness, or lying, in test taking. These dimensions are likely to have played an important role in the process by which the clinicians used the MMPI profiles in their diagnostic judgments in Meehl's experiment (Ganzach, 1995).

Theory

Several studies have investigated nonlinear relationships in the judgments of Meehl's data (Wiggins & Hoffman, 1968; Goldberg, 1971; Einhorn, 1974; Ganzach, 1995), and most of them did indeed find such relationships. (Even Goldberg, 1971, who disputes the earlier conclusions of Einhorn, 1970, 1971, concerning nonlinearity in judgment, concludes that there are nonlinear relationships in these data; see pp. 467–468.) Among these studies, only Wiggins and Hoffman used product and quadratic terms. They found that for 8 of the 29 judges, a model that includes such terms gives a better fit to the data than a model that includes only linear terms (see Wiggins & Hoffman, 1968, Table 3, p. 75). However, their findings shed no light on the issue discussed in the current article, because they neither report the signs of the interaction and the curvilinear terms, nor do they provide any theory about the pattern of the signs of these terms.

Recently, I analyzed the interactive relationships in the judgments of Meehl's data on the basis of various versions of the scatter model (Ganzach, 1995). The hypotheses underlying these analyses were as follows: (a) In Hypothesis 1, there are two important information integration processes associated with the judgments in Meehl's experiment: the integration of interdimension information and the integration of intradimension information. (b) In Hypothesis 2, the integration of the intradimension information is offsetting. First, the interaction between two psychotic scales, P and P' , is such that the relationship between the judgment of psychosis and P becomes more positive as the value of P' decreases (obviously, this relationship is symmetric with regard to P and P' ; that is, it holds when P is replaced by P' , and P' is replaced by P). Second, the interaction between two neurotic scales, N and N' , is such that the relationship between the judgment of neurosis and N becomes more positive as the value of N' decreases. However—because the actual judgment scale in the experiment ranges from least psychotic (i.e., neurotic) to most psychotic—this interaction implies that the relationship between the judgment of psychosis and N become

more negative as the value of N' decreases. (c) In Hypothesis 3, the integration of the interdimension information is offsetting vis-à-vis the judgment of psychosis. That is, the interaction between a psychotic scale, P , and a neurotic scale, N , is such that the relationship between the judgment of psychosis and P becomes more positive as the values of N decreases (this relationship is symmetric with regard to N and P).

These hypotheses are derived from the assumption that the various information integration processes are influenced by confirmatory biases (Klayman & Ha, 1987; Snyder & Campbell, 1980), that lead to an increase in the weight of confirmatory information. First, the integration of the intradimension information is influenced by a dimension-dependent bias. The more neurotic the cue, the larger its weight compared with the other cues of the neurotic dimension; and the more psychotic the cue, the larger its weight in comparison with the other cues of the psychotic dimension. Second, the integration of the interdimension information is influenced by the judgment psychosis being determined primarily by the psychotic dimension (Ganzach, in press-a), and by the tendency of present information to have a larger impact on decision than absent information (Fazio, Sherman, & Herr, 1982).

Analysis

Predictions. Hypothesis 2 implies a negative interaction for pairs of psychotic scales because judges use the higher of the two scores to judge psychosis, and it implies a positive interaction for pairs of neurotic scales, as judges combine the two neurotic scores as evidence against psychosis. Hypothesis 3 implies a negative interaction for pairs of one psychotic scale and one neurotic scale. This prediction can be derived by representing the psychotic dimension as the average of the scores of the psychotic scales and the neurotic dimension as the average of the scores of the neurotic scales.⁵ Using this representation, Hypothesis 3 implies that the coefficient of the product $(P + P')(N + N')$ is negative, which implies that the coefficients of the neurotic-psychotic pairs are also negative.

A model with linear and product terms. This

⁵ Note that for this representation, the scales ought to be expressed on similar scales (e.g., standardized scales). Indeed, in Meehl's data the MMPI scales are standardized (they are represented as test scores).

Table 5
Intercorrelations Among the Product and Quadratic Terms of the Clinical Judgment Model

Term	1	2	3	4	5	6	7	8	9	10
1. P ₁ P ₂	—	0.80	0.90	0.84	0.87	0.69	0.87	0.91	0.45	0.73
2. P ₁ N ₁		—	0.83	0.85	0.68	0.83	0.82	0.65	0.82	0.65
3. P ₁ N ₂			—	0.79	0.90	0.83	0.84	0.79	0.52	0.90
4. P ₂ N ₁				—	0.88	0.90	0.60	0.88	0.80	0.77
5. P ₂ N ₂					—	0.85	0.61	0.94	0.51	0.93
6. N ₁ N ₂						—	0.53	0.71	0.84	0.89
7. P ₁ ²							—	0.62	0.37	0.53
8. P ₂ ²								—	0.44	0.76
9. N ₁ ²									—	0.53
10. N ₂ ²										—

Note. P₁ = Paranoia scale; P₂ = Schizophrenia scale; N₁ = Hysteria scale; N₂ = Psychosthenia scale.

model included all the linear terms (the 11 MMPI scales) and only six product terms. The terms associated with two neurotic scales and two psychotic scales. The reason for the limited number of product terms in the model is that the inclusion of all 55 possible product terms may seriously reduce the power of tests of significance. The six-product terms included in the model corresponded to the two neurotic scales with the larger beta weights (N₁, the Hysteria scale, and N₂, the Psychosthenia scale, a scale that reflects obsessive-compulsive tendencies) and the two psychotic scales with the larger beta weights (P₁, the Paranoia scale, and P₂, the Schizophrenia scale). Thus, the first regression was:

$$Y = \alpha_0 + \sum_{i=1}^4 \alpha_i^N N_i + \sum_{i=1}^5 \alpha_i^P P_i + \sum_{i=1}^2 \alpha_i^T T_i + \beta_1 N_1 N_2 + \beta_2 P_1 P_2 + \beta_3 N_1 P_1 + \beta_4 N_1 P_2 + \beta_5 N_2 P_1 + \beta_6 N_2 P_2. \quad (3)$$

Where Y is the average judgment of psychosis of the 29 clinicians, P_i, N_i, and T_i are the *i*th scale of the psychotic, neurotic, and defensiveness factor, respectively. (Table 5 presents the intercorrelations between the product and quadratic terms that appear in the model.)

Our predictions are that β₁, the coefficient associated with the information integration of the neurotic dimension, is positive; β₂, the coefficient associated with the information integration of the psychotic dimension, is negative; and β₃, β₄, β₅, and β₆, the coefficients associated with the interdimension information integration, are negative. Column 2 of Table 6 presents the six coefficients and their standard errors. The results reveal that only β₁ is significant. The

value of this coefficient is positive, which is consistent with the theory.

A model with linear, product, and quadratic terms. In addition to the terms that appeared in Equation 3, this model also included the quadratic terms of the four scales under investigation:

Table 6
The Nonlinear Terms in the Regression of the Judgment on the 11 MMPI Scales

Term	Only product terms	Only quadratic	Full model
P ₁ ²		.0006 (.0002)	.0022* (.0002)
P ₂ ²		-.0004* (.0001)	.0009* (.0002)
N ₁ ²		.0004 (.0001)	-.0000 (.0000)
N ₂ ²		.0002 (.0001)	.0005 (.0002)
P ₁ × P ₂	-.0002 (.0002)		-.0023* (.0003)
P ₁ × N ₁	-.0002 (.0003)		-.0004 (.0003)
P ₁ × N ₂	-.0003 (.0003)		-.0000 (.0003)
P ₂ × N ₁	-.0005 (.0003)		-.0004 (.0003)
P ₂ × N ₂	-.0002 (.0002)		-.0012* (.0003)
N ₁ × N ₂	.0014* (.0002)		.0013* (.0003)
R ²	.807	.802	.830

Note. Items in parentheses are standard errors. MMPI = Minnesota Multiphasic Personality Inventory; P₁ = Paranoia scale; P₂ = Schizophrenia scale; N₁ = Hysteria scale; N₂ = Psychosthenia scale.

*p < .0001.

$$\begin{aligned}
Y = & \alpha_0 + \sum_{i=1}^4 \alpha_i^N N_i + \sum_{i=1}^5 \alpha_i^P P_i + \sum_{i=1}^2 \alpha_i^T T_i \\
& + \beta_1 N_1 N_2 + \beta_2 P_1 P_2 + \beta_3 N_1 P_1 + \beta_4 N_1 P_2 \\
& + \beta_5 N_2 P_1 + \beta_6 N_2 P_2 \\
& \gamma_1 N_1^2 + \gamma_2 N_2^2 + \gamma_3 P_1^2 + \gamma_4 P_2^2. \quad (4)
\end{aligned}$$

The results of this model are presented in column 3 of Table 6. They indicate that out of the six interactions, three are significant, and their signs are consistent with the theory that the interaction between N_1 and N_2 is significantly positive, the interaction between P_1 and P_2 is significantly negative, and the interaction between N_2 and P_2 is significantly negative. Comparison with the results of the model of Equation 3 reveals that whereas in a model that did not include the quadratic terms only one interaction was significant, in a model that did include these terms, three interactions were significant.⁶ Thus, in this example, the quadratic terms suppress the effects of the product terms.

The results of the model of Equation 4 also indicate that the coefficients of the quadratic terms of the two psychotic scales are positive, which suggests convex relationships between these two scales and the judgment. These relationships are consistent with our theory because they imply that the more psychotic a cue, the larger its weight.

Misleading curvilinear terms. A comparison between the full model (Equation 4) and a partial model that includes only linear and quadratic terms can provide a demonstration of misleading curvilinear terms. The results of this latter model are presented in column 4 of Table 6. While the full model revealed two significant quadratic terms, both consistent with the theory, the partial model revealed one significant quadratic term, the quadratic term of P_2 , whose sign is, in disagreement with the theory, negative. The discussion above suggests that the observed curvilinear relationship associated with P_2 in the partial model is misleadingly concave, and the true convex curvilinear relationship emerges only from the full model.

Discussion: On Estimating Interactions in Regression Models When the Independent Variables Are Correlated

This article demonstrates the importance of including quadratic terms when interactions are estimated. In particular, it shows that without quadratic terms, a nonsignificant interaction may be observed in the

presence of a strong true interaction, and that an observed interaction may be positive when the true interaction is negative (misleading interaction).

This article also demonstrates the importance of including product terms when quadratic terms are estimated, showing that without including product terms, observed nonsignificant quadratic terms may occur in the presence of true curvilinear relationships. It also shows that without product terms, a concave relationship may be observed in the presence of a strong true convex relationship, and similarly, a convex relationship may be observed in the presence of a strong true concave relationship (misleading curvilinear terms).

In an earlier article that dealt with these issues, Lubinski and Humphreys (1990) suggested that the failure to include quadratic terms when independent variables are correlated may lead to Type I error in testing hypotheses about interaction. But the current research indicates that not including quadratic terms may also lead to Type II error.⁷ That is, it is possible that previous attempts to detect (theoretically meaningful) interactions in the presence of multicollinearity failed because researchers did not include quadratic terms in their equations. Thus, a critical look at the results of tests of interaction in the literature should take into account that multicollinearity may increase both the probability of Type I error, and the probability of Type II error. Whether Type I or Type II errors are likely to occur depends on the type of interaction and the form of the nonlinear relationship between independent and dependent variables. Table 7 presents the conditions in which Type I and Type II errors occur when interaction is examined in a model that does not include quadratic terms. Type I error occurs when the true relationship is quadratic but not interactive, and Type II occurs when the true quadratic relationship is positive (negative) and the true interactive relationship is negative (positive). The

⁶ Note that the difference between the values of the two interactions that were "revealed" in the full model and their values in the partial model is quite substantial; the former are about 6 to 12 times as high as the latter (compare columns 2 and 4 of Table 5).

⁷ Lubinski and Humphreys (1990) also discuss Type II error in detecting interaction effects. But this discussion concerns the (un)reliability of the product term and is unrelated to the issue of the relationship between quadratic and product terms in testing for interaction effects.

Table 7
The Conditions in Which Type I and Type II Errors Occur When Interaction is Examined in a Model That Does Not Include Quadratic Terms

True interaction term	True quadratic term		
	Positive (convex)	Zero	Negative (concave)
Positive			Type II
Zero	Type I		Type I
Negative	Type II		

empty cells in the table, except the central cell, correspond to the Type I and Type II error when the estimated model includes quadratic, but not interaction, terms.

A related issue is the small effect size of interactions that is usually found in nonexperimental work and has distributed many researchers. This small effect size may also be partially due to the failure to include quadratic terms. For example, in a recent article, McClelland and Judd (1993) showed that a small interaction effect was associated with an interaction that was theoretically as large as it can be (p. 383). But McClelland and Judd did not include quadratic terms in their equation. Because there was considerable multicollinearity in the data, the inclusion of these terms may have led to an increase in the interaction effect. Thus, in addition to structural factors that lead to difficulties of detecting interactions in nonexperimental data (see McClelland & Judd, 1993, for a thorough discussion of these factors), not including quadratic terms may be an additional important reason for these difficulties.

The question whether quadratic terms should be introduced into the regression when hypotheses about interactions are examined has been receiving a close scrutiny lately. Although almost all researchers agree that attention should be paid to quadratic terms when such hypotheses are examined, the views regarding this question range from Lubinski and Humphreys's (1990) "conservative" opinion that quadratic terms should always be examined using a stepwise method, to Shepperd (1991) and Aiken and West (1991) who caution against introducing quadratic terms without sufficient theoretical justification. Recently, Cortina (1993) suggested an even more conservative approach than that of Lubinski and Humphreys's. He suggested that quadratic terms should be introduced into the regression prior to the introduction of product terms; that is, that hypotheses about interactions should be

tested by comparing the incremental variance of a model that includes linear, product, and quadratic terms over a model that includes linear and quadratic terms.

Although the Cortina argument that this approach should be used because there is no loss of power in introducing quadratic terms into the regression is problematic (Ganzach, in press-b), there is, in my view, merit in the approach itself. There are two main reasons for this. First, psychological theories are usually associated with a conditionally monotone, and not with a conditionally linear, relationship between independent and dependent variables.⁸ For example, we expect the child's educational expectations to increase monotonically, and not necessarily linearly, with the education of the parent. Though there are numerous forms of a monotone relationship, a quadratic function may be a good approximation for many of those which are likely to occur in psychological research (Cohen & Cohen, 1983, pp. 225–229).⁹ Thus, a model that includes quadratic terms is a better representation of underlying theories than one that includes only linear terms. Second, psychological measurements are usually associated with a monotone, rather than linear, relationship between the true score of the variable and the measure of this variable (e.g., Krantz & Tversky, 1971). This may result in significant quadratic terms in a regression even if the true relationships are linear (Busemeyer & Jones, 1983).

In summary, because of basic considerations concerning the level of measurement and the form of the

⁸ For example, Lubinski and Humphreys's (1990) null hypothesis was "The genesis of brilliant mathematical accomplishment is simply [the result of] an extraordinary level of quantitative ability" (p. 386). This is an hypothesis about monotonic, and not about linear, relationships. Nevertheless the null hypothesis which is tested by Lubinski and Humphreys's is a linear hypothesis. It could be argued that the spurious interaction that Lubinski and Humphreys's demonstrated is simply the result of misinterpretation of the null hypothesis.

⁹ For example, higher order polynomials are "rarely useful in behavioral research" (Cohen & Cohen, 1983, p. 229). One reason is that linear and quadratic terms are highly correlated with other possible nonlinear terms that may explain additional variance (e.g., whereas the linear term is highly correlated with any terms whose powers is odd, the quadratic term is highly correlated with terms whose power is even; see Budescu, 1980), thus adding additional nonlinear terms is not likely to add significantly to the explained variance.

relationship between independent and dependent variables, which are unrelated to any specific multiplicative theory tested by the researcher, quadratic terms should be introduced into the model, even if the theory being tested is about interaction. This recommendation is similar to the recommendation to examine linear terms even if the theory being tested is multiplicative. For example, Cronbach (1987) argues that even if there is a strong interaction hypothesis, the product term should be tested only after the linear variance has been partialled out: "The reader who starts with another presupposition or uncertainty, is unlikely to be persuaded that the conclusion from the analysis [that consider only an interaction effect], whether positive or negative, is valid" (p. 417). Similarly, readers who start with an uncertainty about the linearity of the relationships between independent and dependent variables and about the linearity of the relationships between true scores and observed measures are unlikely to be persuaded by tests of interaction hypotheses that do not partial out curvilinear variance. (See also Anderson, 1982, for a related discussion regarding tests of multiplicative rules in information integration theory.)

Each of the two studies reported in this paper is a good example for the priority that should be given to the curvilinear over the interaction terms. In both studies the basic underlying theory concerned interaction, rather than curvilinear, relations, though strong curvilinear relations were nevertheless found. Other examples are presented by Ganzach and Czaczkes (1996), who reanalyzed a number of published studies examining hypotheses about interactive relationships (e.g., Ganzach, 1995; Ganzach & Czaczkes, 1995). In addition to the interactive relationships that were predicted by the theory, they found in many of these studies curvilinear relationships that were not predicted by the theory (and therefore not examined in the original analyses). My previous analysis of Meehl's data (Ganzach, 1995) is a case in point. In this analysis, no curvilinear terms were included because there was no theory suggesting considering these terms. Nevertheless, adding curvilinear terms to the models used in this previous analysis does increase the explained variance significantly.

So far, I have treated the relationship between quadratic and product terms in the context of perfectly reliable variables. In both the substantive examples and the subsequent discussion, I have assumed that there is no error in the variables. However, when the independent variables are measured with error, the

issue of differential reliability of the product and quadratic terms arises. In particular, because product terms are more reliable than quadratic terms, when the correlation between the true scores of the independent variables is high, and both quadratic and product terms are tested, the estimated model could be multiplicative, even though the true model is quadratic (MacCullum & Marr, 1995). This situation also calls for a conservative approach of introducing the quadratic terms into the regression prior to the product terms.

There are, of course, costs in introducing quadratic terms prior to the introduction of the product terms. Loss of power and unstable regression coefficients are the most important. These problems may be compounded by the relatively small sample sizes that are usually available—much smaller than the sample sizes used in the current article. Further work is necessary to assess the extent of these problems.¹⁰ However, in my view, the practice of examining for curvilinear relationships when interaction hypotheses are tested is particularly important because people in general, and researchers in particular, are both biased against curvilinear hypotheses and have a tendency to overhypothesize interactive relationships. Elsewhere (Ganzach, in press-a), I have demonstrated these phenomena by comparing models of the judgments to models of the corresponding criteria. The results of these studies indicated that although the judgments are highly interactive, the criteria are not. For example, the interactive effects that characterize clinical psychologists' judgment of pathology do not characterize the actual pathology, and the interactive effects that characterize teenagers' educational expectations do not characterize their educational attainment.¹¹ In-

¹⁰ Elsewhere (Ganzach, in press-b), I conducted some simulations to study the effect of adding quadratic terms on the probability of detecting an interaction when the true model is multiplicative. These simulations indicate that, for the typical situations encountered in psychological research (e.g., when the variance attributed to the interaction is 3%, the multicollinearity is .3, and the sample size is 200), the loss of power is rather small.

¹¹ On the other hand, curvilinear effects characterize both the judgment and the criteria. For example, both the relationship between the parents' education and the educational expectations of the child, and the relationship between the parents' education and the educational attainment of the child are convex. Similarly, the curvilinear relationships that characterize the judgment of pathology tend also to characterize the actual pathology.

deed, even a casual examination of the academic literature reveals that biases toward interactive hypotheses and against curvilinear hypotheses are common among researchers in the social sciences: The nonlinear relationships that are examined in the literature are almost always interactive and rarely curvilinear (see Cortina, 1993, for a survey of this phenomenon). This may lead not only to Type I error (Lubinski & Humphreys, 1990), but it also may lead—as the current article demonstrates—to Type II errors and even misleading significant results.

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